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The decays $K \to \pi\ell^+\ell^-$ beyond leading order in the chiral expansion*

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Abstract: We present a model-independent analysis of $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$ decays, including $K \to 3\pi$ unitarity corrections and a general decomposition of the dispersive amplitude. From the existing data on $K^+ \to \pi^+e^+e^-$ we predict the ratio $R = B(K^+ \to \pi^+\mu^+\mu^-)/B(K^+ \to \pi^+e^+e^-)$ to be larger than 0.23, in slight disagreement with the recent measurement $R = 0.167 \pm 0.036$. Consequences for the $K^\pm \to \pi^\pm e^+e^-$ charge asymmetries and for the $K_L \to \pi^0e^+e^-$ mode are also discussed.

Keywords: CP violation, Kaon Physics, Chiral Lagrangians.

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JHEP08(1998)004

Contents

JHEP08(1998)004

1. Introduction

Radiative nonleptonic kaon decays continue to provide interesting information on the structure of the weak interactions at low energies. Among them, the flavour–changing neutral current (FCNC) transitions $K \to \pi \ell^+ \ell^-$, induced at the one–loop level in the Standard Model, are well suited to explore its quantum structure and, possibly, its extensions [1, 2]. On the experimental side, the high–precision measurements already achieved at Brookhaven (AGS) [3] and Fermilab (KTeV) [4], and foreseen in the near future at Frascati (KLOE) [5], call for a thorough theoretical investigation.

The $K^+ \to \pi^+ \ell^+ \ell^-$ and $K_S \to \pi^0 \ell^+ \ell^-$ ($\ell = e, \mu$) channels, expected to be dominated by long–distance dynamics through one–photon exchange ($K \to \pi \gamma^*$), were studied at leading order in the chiral expansion in Ref. [1]. Once the unknown combination of local terms arising in this framework is fixed by comparison with $K^+ \to \pi^+ e^+ e^-$ data [6], the process $K^+ \to \pi^+ \mu^+ \mu^-$ can be predicted without further assumptions.[1] Recently, this last mode has been observed [7], yielding a value for $R = \Gamma(K^+ \to$

[1] The neutral channels (still unmeasured) require additional model–dependent assumptions as discussed in Sect. 3.

$\pi^+\mu^+\mu^-)/\Gamma(K^+ \to \pi^+e^+e^-)$ that is $\sim 2\,\sigma$ below the leading–order prediction. This experimental result has motivated the present study of $K \to \pi\gamma^*$ form factors beyond the leading order in the chiral expansion. We have carried out this program by including unitarity corrections from $K \to \pi\pi\pi$ together with the most general polynomial structure consistent with the chiral expansion up to $\mathcal{O}(p^6)$. In this way we perform a model–independent analysis of the existing $K^+ \to \pi^+\ell^+\ell^-$ data leading to a consistent fit of both rate and dilepton invariant–mass spectrum of the electron channel. On the other hand, we demonstrate that the persisting discrepancy in the ratio R cannot be accommodated in the Standard Model.

Besides this phenomenological analysis, we present a general discussion of theoretical predictions for the polynomial part of the $K^+(K_S) \to \pi^+(\pi^0)\gamma^*$ form factors. We show that it is very difficult at present to estimate the branching ratios of the $K_S \to \pi^0\ell^+\ell^-$ modes without strong model–dependent assumptions. More accessible are the dilepton invariant–mass spectrum and, correspondingly, the ratio $\Gamma(K_S \to \pi^0\mu^+\mu^-)/\Gamma(K_S \to \pi^0e^+e^-)$.

We conclude our analysis by discussing the impact of the long–distance $K \to \pi\gamma^* \to \pi e^+e^-$ transitions for estimating CP–violating observables. In particular, we analyse the possibilities for disentangling direct and indirect CP–violating components of the $K_L \to \pi^0e^+e^-$ amplitude. Moreover, we estimate the effect of the unitarity corrections on the charge asymmetry of $K^\pm \to \pi^\pm e^+e^-$ decays.

The plan of the paper is the following: in the next section we present the model–independent analysis of $K \to \pi\gamma^* \to \pi\ell^+\ell^-$ transitions. Section 3 is devoted to explore the physics behind the polynomial coefficients of the $K \to \pi\gamma^*$ form factors. We turn to a discussion of CP–violating observables in Section 4. Our main conclusions are summarized in the last section.

JHEP08(1998)004

2. Model-independent analysis

2.1. Kinematics

The FCNC transitions $K \to \pi\ell^+\ell^-$ ($\ell = e,\mu$) are dominated by single virtual photon exchange ($K \to \pi\gamma^* \to \pi\ell^+\ell^-$) if allowed by CP symmetry. This is the case for K^+ and K_S decays where the amplitude is determined by an electromagnetic transition form factor in the presence of the nonleptonic weak interactions:

$$i\int d^4x e^{iqx}\langle\pi(p)|T\left\{J^\mu_{\rm elm}(x)\mathcal{L}_{\Delta S=1}(0)\right\}|K(k)\rangle = \frac{W(z)}{(4\pi)^2}\left[z(k+p)^\mu - (1-r_\pi^2)q^\mu\right], \quad (2.1)$$

$$k^2 = M_K^2\,, \quad p^2 = M_\pi^2\,, \quad q = k-p\,, \quad z = q^2/M_K^2\,, \quad r_\pi = M_\pi/M_K\,,$$

where $\mathcal{L}_{\Delta S=1}$ is the strangeness changing nonleptonic weak Lagrangian and $J^\mu_{\rm elm}$ is the electromagnetic current. The dynamics of the decays is completely specified by the invariant functions $W_+(z)$ and $W_S(z)$. As dictated by gauge invariance, these functions vanish to lowest order in the low–energy expansion [1]. To account for this chiral suppression we have pulled out a factor $1/(4\pi)^2$ in the definition (2.1). Since the Cabibbo angle is approximately compensated by the nonleptonic octet enhancement, the natural magnitude of $W(z)$ is expected to be $G_F M_K^2$.

With these conventions[2] the decay amplitude takes the form

$$A\Big(K(k)\to\pi(p)\ell^+(p_+)\ell^-(p_-)\Big) = -\frac{e^2}{M_K^2(4\pi)^2}W(z)(k+p)^\mu\bar{u}_\ell(p_-)\gamma_\mu v_\ell(p_+)\,. \qquad (2.2)$$

The spectrum in the dilepton invariant mass is then given by

$$\frac{d\Gamma}{dz} = \frac{\alpha^2 M_K}{12\pi(4\pi)^4}\,\lambda^{3/2}(1,z,r_\pi^2)\sqrt{1-4\frac{r_\ell^2}{z}}\left(1+2\frac{r_\ell^2}{z}\right)|W(z)|^2\,, \qquad (2.3)$$

with $r_\ell = m_\ell/M_K$, $4r_\ell^2 \le z \le (1-r_\pi)^2$ and $\lambda(a,b,c) = a^2+b^2+c^2-2(ab+ac+bc)$.

2.2. Form factors

The form factors $W_i(z)$ $(i=+,S)$ are analytic functions in the complex z–plane cut along the positive real axis. The cut starts at $z=4r_\pi^2$ with the two–pion threshold. For the small dilepton masses occurring in the decays, one expects the $\pi^+\pi^-$ intermediate state to play the dominant role in the dispersion relations for $W_i(z)$. The contribution of higher–mass intermediate states can be described by a low–order polynomial in z. This is known to be a very good approximation for the K^+K^- contribution, for instance [1].

We thus decompose the form factors $W_i(z)$ as

$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)\,, \qquad (2.4)$$

where $W_i^{\pi\pi}(z)$ denotes the contribution from the two–pion intermediate state. To leading nontrivial order in the chiral expansion [1], the $W_i^{\rm pol}(z)$ are constants (but for a negligible contribution due to the kaon loop). The functions $W_i^{\pi\pi}(z)$ can be calculated from the diagram shown in Fig. 1, where the $K\to 3\pi$ vertex is taken from the leading nonleptonic weak Lagrangian of $\mathcal{O}(p^2)$.

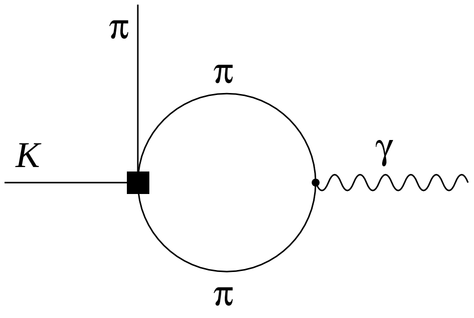

Figure 1: $K\to 3\pi$ contribution to the effective $K\to\pi\gamma^*$ vertex.

Here we go one step further. We use the physical $K\to 3\pi$ amplitude expanded up to $\mathcal{O}(p^4)$ (as required by present $K\to 3\pi$ data) and include the electromagnetic form

[2] In Ref. [2] the form factor $V(z) = W(z)/G_8 M_K^2$ was used instead of $W(z)$. Here we prefer to avoid introducing the coupling constant G_8 in the model–independent analysis. We use the occasion to point out that there are two misprints in eq. (4.27) of [2]; the correct formula is given by (2.3).

JHEP08(1998)004

factor $F(z)$ (normalized to $F(0) = 1$) for $\pi^+\pi^- \to \gamma^*$ to first nontrivial order. The relevant $K \to 3\pi$ amplitudes are expanded as [8]

$$
\begin{aligned}
A(K^+ \to \pi^+\pi^+\pi^-) &= 2a_c + (b_c + b_2)Y + 2c_c(Y^2 + X^2/3) + (d_c + d_2)(Y^2 - X^2/3)\,, \\
A(K_S \to \pi^+\pi^-\pi^0) &= \frac{2}{3}b_2 X - \frac{4}{3}d_2 XY\,,
\end{aligned}
\tag{2.5}
$$

with

$$
s_i = (k - p_i)^2\,, \qquad s_0 = \frac{1}{3}(s_1 + s_2 + s_3)\,, \qquad X = \frac{s_1 - s_2}{M_\pi^2}\,, \qquad Y = \frac{s_3 - s_0}{M_\pi^2}\,, \tag{2.6}
$$

where p_i denote the pion momenta.[3] To the same level of accuracy, the electromagnetic form factor is $F(z) = 1 + z/r_V^2$ with $r_V^2 = M_V^2/M_K^2 \simeq 2.5$. In accordance with chiral counting, the polynomial in (2.4) is assumed to have the general form

$$
W_i^{\rm pol}(z) = a_i + b_i z \qquad (i = +, S)\,. \tag{2.7}
$$

Up to a linear polynomial of this type, the dispersion integral corresponding to Fig. 1 is unambiguously calculable with the result

$$
W_i^{\pi\pi}(z) = \frac{1}{r_\pi^2}\left[\alpha_i + \beta_i \frac{z - z_0}{r_\pi^2}\right] F(z)\ \chi(z)\,, \tag{2.8}
$$

where $z_0 = 1/3 + r_\pi^2$. The one–loop function

$$
\chi(z) = \frac{4}{9} - \frac{4r_\pi^2}{3z} - \frac{1}{3}\left(1 - \frac{4r_\pi^2}{z}\right)G(z/r_\pi^2) \tag{2.9}
$$

$$
G(z) = \begin{cases} \sqrt{4/z - 1}\,\arcsin\left(\sqrt{z}/2\right) & z \le 4 \\ -\frac{1}{2}\sqrt{1 - 4/z}\left(\ln \dfrac{1 - \sqrt{1 - 4/z}}{1 + \sqrt{1 - 4/z}} + i\pi\right) & z \ge 4 \end{cases}
$$

satisfies $\chi(0) = 0$. In terms of the $K \to 3\pi$ parameters in (2.5), we find

$$
\alpha_+ = -(b_c + b_2)\,, \qquad \beta_+ = 2(d_c + d_2)\,, \qquad \alpha_S = \frac{4}{3}b_2\,, \qquad \beta_S = -\frac{8}{3}d_2\,. \tag{2.10}
$$

The total form factor, given by

$$
W_i(z) = G_F M_K^2 (a_i + b_i z) + W_i^{\pi\pi}(z)\,, \tag{2.11}
$$

is expected to be an excellent approximation to the complete form factor of $\mathcal{O}(p^6)$. The polynomial piece has the most general form that can occur to this order, with a priori unknown low–energy constants contributing to the a_i, b_i. The main assumption

[3] The subscript 3 indicates the odd–charge pion, i.e. π^- in $K^+ \to \pi^+\pi^+\pi^-$ and π^0 in $K_S \to \pi^+\pi^-\pi^0$.

JHEP08(1998)004

b_c	b_2	d_c	d_2
24.5 ± 0.3	-3.9 ± 0.4	-1.6 ± 0.3	0.2 ± 0.5

Table 1: Experimental values of $K \to 3\pi$ amplitudes [9] contributing to the $W_i^{\pi\pi}(z)$. All entries are in units of 10^{-8}.

underlying the form factor (2.11) is that all other contributions to the dispersion integral except the two–pion intermediate state can be well approximated by a linear polynomial for small values of z.

Using the central values of the $K \to 3\pi$ parameters [9] in Table 1, we can now calculate the branching ratios for $K^+ \to \pi^+\ell^+\ell^-$ and $K_S \to \pi^0\ell^+\ell^-$ in terms of the corresponding parameters:

$$\begin{aligned}
B(K^+\to\pi^+e^+e^-) &= \Big[0.14 - 3.23a_+ - 0.88b_+ + 59.2a_+^2 + 16.0a_+b_+ + 1.73b_+^2\Big]\times 10^{-8}\,,\\
B(K^+\to\pi^+\mu^+\mu^-) &= \Big[1.13 - 19.2a_+ - 6.32b_+ + 116a_+^2 + 67.3a_+b_+ + 10.3b_+^2\Big]\times 10^{-9}\,,\\
B(K_S\to\pi^0e^+e^-) &= \Big[0.01 - 0.76a_S - 0.21b_S + 46.5a_S^2 + 12.9a_Sb_S + 1.44b_S^2\Big]\times 10^{-10};\\
B(K_S\to\pi^0\mu^+\mu^-) &= \Big[0.07 - 4.52a_S - 1.50b_S + 98.7a_S^2 + 57.7a_Sb_S + 8.95b_S^2\Big]\times 10^{-11}\,. \qquad (2.12)
\end{aligned}$$

With a_+ and a_S expected to be of $\mathcal{O}(1)$, we observe that the amplitudes and branching ratios are actually dominated by the polynomial parts. $W_i^{\pi\pi}(z)$ in (2.8) contributes to the rate mainly through its interference with (2.7). In fact, this really only applies to the K^+ mode because the two–pion contribution is practically negligible for the neutral decay, due to the strong suppression of the $K_S \to \pi^+\pi^-\pi^0$ amplitude.

2.3. Comparison with experiment

In the Brookhaven experiment BNL-E777 [6] both spectrum and branching ratio of the decay $K^+ \to \pi^+e^+e^-$ were measured. They fit a spectrum with two parameters C and λ defined through

$$\frac{d\Gamma}{dM_{ee}} = \frac{C}{8}\, M_{ee}\, M_K^3\, \lambda^{3/2}(1, r_\pi^2, \frac{M_{ee}^2}{M_K^2}) \left(1 + \lambda\,\frac{M_{ee}^2}{M_\pi^2}\right)^2 , \qquad (2.13)$$

where M_{ee} is the invariant mass of the lepton pair ($M_{ee} = M_K\sqrt{z}$), obtaining

$$\lambda = 0.105 \pm 0.035 \pm 0.015\,, \qquad (2.14)$$

$$B(K^+\to\pi^+e^+e^-) = (2.75 \pm 0.23 \pm 0.13)\times 10^{-7}\,. \qquad (2.15)$$

However, in order to subtract the background due to the process $K^+ \to \pi^+\pi^0$, $\pi^0 \to e^+e^-\gamma$, they make a cut in the dilepton invariant mass and consider only $K^+ \to \pi^+e^+e^-$

JHEP08(1998)004

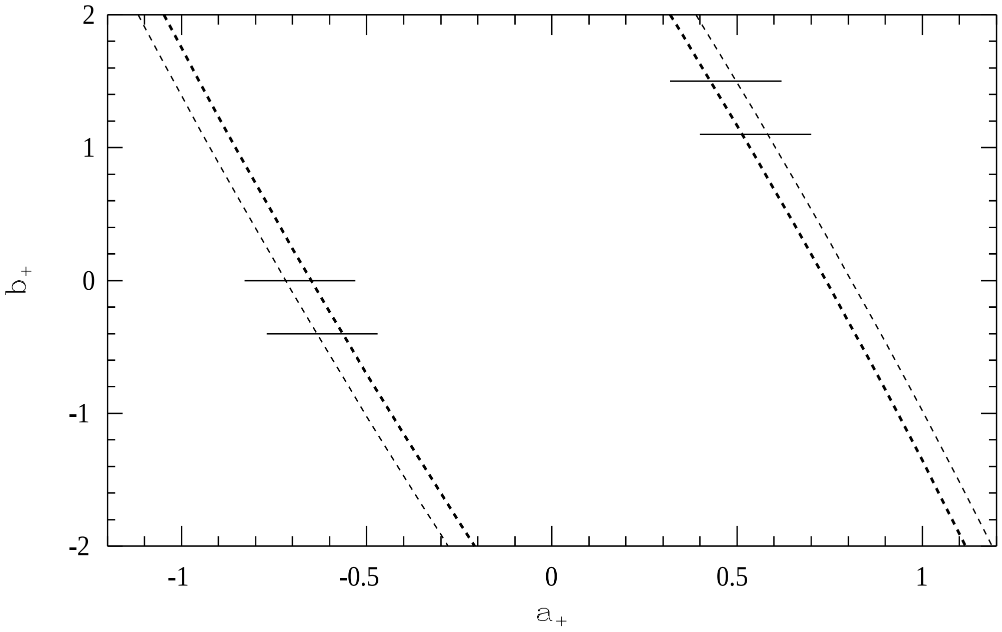

Figure 2: Allowed domains in the a_+–b_+ plane (inside the dashed curves) from the experimental branching ratio (2.16). The bounds on b_+ from the measured slope (2.14) are also shown (full lines).

events with $M_{ee} > 0.150$ GeV. Thus the branching ratio actually measured is given by[4]

$$B(K^+ \to \pi^+ e^+ e^-)\,|_{\rm cut} \;=\; (1.81 \pm 0.17) \times 10^{-7} \tag{2.16}$$

and the result in (2.15) includes a theoretical extrapolation to the low M_{ee} region.

The branching ratio (2.16) can be translated into allowed domains for the parameters a_+ and b_+ as shown in Fig. 2. For a generous range $|b_+| \le 2$, two branches of solutions are in principle possible with opposite signs of a_+.

Unlike the rate, the spectrum is very sensitive to b_+. The bounds on b_+ from the experimental spectrum are also indicated in Fig. 2. Let us try to understand the allowed values of a_+ and b_+ with the help of the amplitude (2.11). The real part of the two–pion contribution, Re $W_+^{\pi\pi}(z)$, is a monotonically decreasing function of z in the physical region. Assuming first $b_+ = 0$, as would be the case at $\mathcal{O}(p^4)$ (except for the tiny contribution from the kaon loop), we infer that a_+ must be negative to reproduce the experimentally observed positive slope (2.14). In fact, the spectrum prefers a somewhat bigger slope requiring $b_+/a_+ > 0$. This is made more explicit in Fig. 3, where the theoretical spectrum (or rather the square of the form factor) is confronted with the experimental slope.

[4]We have extracted this value from (2.14) and (2.15) using (2.13) for the spectrum. For a conservative estimate of the error, we have scaled down the error given in (2.15).

JHEP08(1998)004

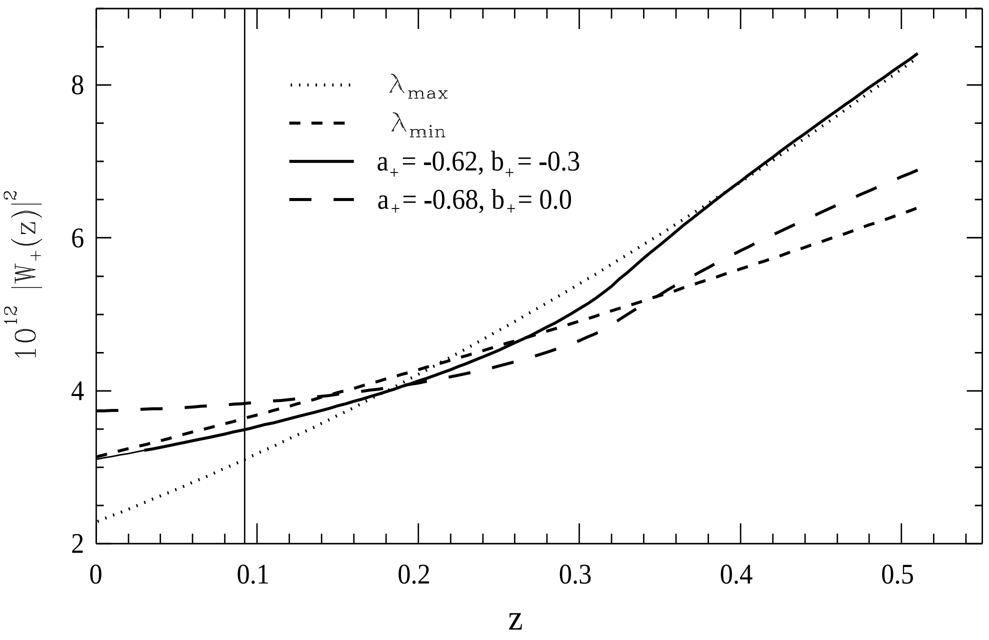

Figure 3: Comparison between different shapes of the $K^+ \to \pi^+ e^+ e^-$ form factor for $a_+ < 0$. The dotted and short–dashed lines correspond to the $\pm 1\sigma$ values of the λ parameter in (2.14). The solid and long–dashed lines correspond to the theoretical prediction for two sets of values of a_+ and b_+. The comparison is to be applied for $M_{ee} > 0.150$ GeV (vertical line) as measured by experiment [6].

At $\mathcal{O}(p^4)$ only negative values of a_+ are allowed [6]. However, in general there can be a second branch of solutions with positive a_+. Since in this case the slope is negative for $b_+ = 0$, a sufficiently large b_+ ($b_+/a_+ > 1$) is necessary to reproduce the observed spectrum. Analogously to Fig. 3, we exhibit the square of the form factor in Fig. 4 for two sets of a_+ and b_+ corresponding to the $\pm 1\sigma$ values of the measured slope.

As we shall discuss in the next section, the second branch of solutions with positive a_+ is disfavoured from a theoretical point of view. Indeed, naive chiral counting would suggest that the coefficient b_+, which only arises at $\mathcal{O}(p^6)$, should be smaller in magnitude than the leading coefficient a_+.

For the present analysis we do not discard the second type of solutions right away but confront the theoretical amplitude with another piece of experimental information that has recently become available through the measurement of the muonic decay mode of K^+. The Brookhaven experiment of Adler et al. [7] has measured a branching ratio

$$B(K^+ \to \pi^+ \mu^+ \mu^-) = (5.0 \pm 0.4(\text{stat}) \pm 0.7(\text{syst}) \pm 0.6(\text{th})) \times 10^{-8} \tag{2.17}$$

which, together with the results from [6], implies a muon/electron ratio

$$R = \frac{B(K^+ \to \pi^+ \mu^+ \mu^-)}{B(K^+ \to \pi^+ e^+ e^-)} = 0.167 \pm 0.036\,. \tag{2.18}$$

JHEP08(1998)004

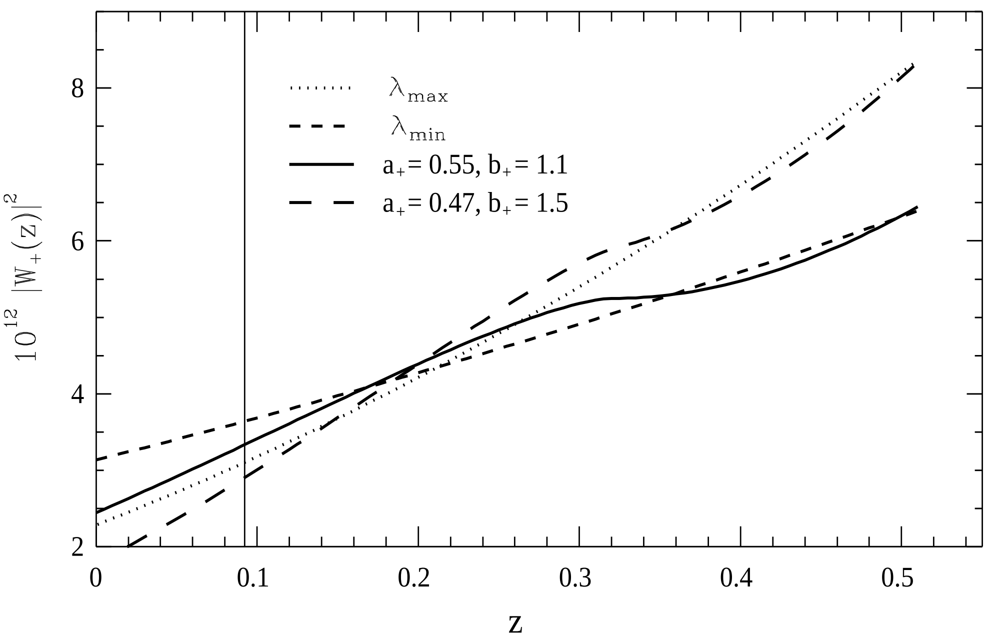

Figure 4: Comparison between different shapes of the $K^+ \to \pi^+ e^+ e^-$ form factor for $a_+ > 0$ (as in Fig. 3).

a_+	b_+	10^8 B$(K^+ \to \pi^+\mu^+\mu^-)$	R
-0.68	0.0	6.78	0.228
-0.62	-0.3	7.29	0.258
0.55	1.1	7.19	0.265
0.47	1.5	7.89	0.309

Table 2: $B(K^+ \to \pi^+\mu^+\mu^-)$ and the ratio R as a function of a_+ and b_+.

Can we use this experimental input to distinguish between the two branches in the $a_+ - b_+$ plane? Looking first at the solutions with $a_+ < 0$, we find that R is a monotonically increasing function of $|b_+|$ for all allowed values in the left branch of Fig. 2 and it is therefore always bigger than 0.23 (obtained for $b_+ = 0$). At first sight the second branch looks more promising for understanding the experimental result (2.18). In fact, for $b_+ = 0$ the ratio R is small enough [1] to agree with (2.18). However, as Fig. 2 shows, the experimental spectrum for the electronic decay channel requires $b_+ > a_+$. Since R is an increasing function of b_+ for $a_+ > 0$, the resulting values of R turn out to be even larger than in the previous case. Therefore, the theoretically unattractive solutions with positive a_+ cannot explain the small μ/e ratio either. For the values of a_+, b_+ used in Figs. 3, 4 the results are collected in Table 2.

On the basis of this model–independent analysis, we conclude that the central values of the μ/e ratio [7] and the slope of the electron spectrum [6] together are not consistent. The $\sim 2\sigma$ discrepancy can be due to a statistical fluctuation or could be an indication of

JHEP08(1998)004

peculiar non–standard physics. We stress that the inconsistency is essentially unrelated to the chiral expansion but only depends on the assumption that both channels are dominated by the same $K \to \pi\gamma^*$ form factor. Indeed, a form factor $W_+(z)$ rising with z as indicated by $K^+ \to \pi^+ e^+ e^-$ data implies $R > R_{\text{phase-space}} = 0.196$ [1].

3. Theoretical ideas on a_i and b_i

The leading $\mathcal{O}(p^4)$ predictions of a_i and b_i are given by

$$\begin{aligned}
a_+^{(4)} &= \frac{G_8}{G_F}\,(1/3 - w_+)\,, \\
a_S^{(4)} &= -\frac{G_8}{G_F}\,(1/3 - w_S)\,, \\
b_+^{(4)} &= -\frac{G_8}{G_F}\,\frac{1}{60}\,, \\
b_S^{(4)} &= \frac{G_8}{G_F}\,\frac{1}{30}\,,
\end{aligned} \tag{3.1}$$

in terms of the usual parameters G_8 and w_i [1]. Local contributions to the b_i are forbidden at $\mathcal{O}(p^4)$ and the small values in (3.1) are generated by the expansion of the kaon–loop function. Sizable corrections to the b_i are expected at $\mathcal{O}(p^6)$ where local terms are allowed. However, since local terms contribute to the a_i already at $\mathcal{O}(p^4)$ (w_i terms), naive chiral counting would suggest $b_i/a_i \sim \mathcal{O}(p^6)/\mathcal{O}(p^4) < 1$. The solution with both a_+ and b_+ negative discussed in the previous section satisfies this expectation.

The expressions of w_+ and w_S in terms of the $\mathcal{O}(p^4)$ low–energy couplings N_i [10], L_9 [11] are

$$\begin{aligned}
w_+ &= \frac{64\pi^2}{3}\,[\,N_{14}^r(\mu) \;-\; N_{15}^r(\mu) \;+\; 3L_9^r(\mu)\,] + \frac{1}{3}\ln\left(\frac{\mu^2}{M_K M_\pi}\right), \\
w_S &= \frac{32\pi^2}{3}\,[\,2N_{14}^r(\mu) \;+\; N_{15}^r(\mu)\,] + \frac{1}{3}\ln\left(\frac{\mu^2}{M_K^2}\right).
\end{aligned} \tag{3.2}$$

Unfortunately, the values of the N_i^r are not known and to make definite predictions we need to rely on model–dependent assumptions. For this purpose, it is interesting to note that while the combinations of low–energy constants appearing in w_+ and w_S (or a_+ and a_S) depend separately on the renormalization scale μ, the combination occurring in $a_+ + a_S$ does not. As originally noted in [1], this scale independence might be related to the structure of the effective four–fermion Hamiltonian relevant for $K \to \pi\ell^+\ell^-$. Indeed, as we will discuss in the next section, there is only one dimension–six operator giving a non–vanishing contribution to the a_i at leading order: $Q_{7V} = \bar{s}\gamma^\mu(1-\gamma_5)d\bar{\ell}\gamma_\mu\ell$. Due to the octet structure of the $\bar{s}\gamma^\mu d$ current, this operator does not affect the combination $a_+ + a_S$. Actually, the cancellation of this short–distance

JHEP08(1998)004

contribution holds not only for $a_+ + a_S$ but (in the limit of isospin conservation) for $A(K^+ \to \pi^+\ell^+\ell^-) + A(K_S \to \pi^0\ell^+\ell^-)$ in general, that would thus be completely determined by low–energy dynamics. As a consequence, we find it more reliable to predict the sum rather than the separate expressions of $W_+^{\rm pol}$ and $W_S^{\rm pol}$ using a low–energy model. In the following we shall employ the Vector Meson Dominance (VMD) hypothesis to estimate $W_+^{\rm pol} + W_S^{\rm pol}$.

The vector meson contributions to the low–energy constants of the $\mathcal{O}(p^4)$ weak Lagrangian have been discussed in [10, 12]. Employing the vector field formulation of ref. [12], the result for $W_+^{\rm pol} + W_S^{\rm pol}$ is independent of the factorization hypothesis and can be specified in terms of a single unknown parameter η_V

$$W^{(4)}_{+,V} + W^{(4)}_{S,V} = \frac{G_8}{G_F}\left[16\pi^2 f_V^2 \,(\,2\eta_V - 1\,) + \frac{1}{3}\ln\left(\frac{M_\pi}{M_K}\right)\right], \tag{3.3}$$

where $|f_V| \simeq 0.20$, as obtained from $\Gamma(\rho^0 \to e^+e^-)$. The logarithmic term in (3.3) is a residual effect of the loop amplitudes, whereas the term proportional to f_V is the local vector meson contribution. The η_V parameter is not known but is expected to be in the range $0 \lesssim \eta_V \lesssim 1$; in principle it is measurable in other processes [12]. Note that for typical values of η_V (but for $\eta_V \simeq 1/2$) the local vector meson contribution is dominant. A relation equivalent to (3.3) was obtained in [10] under the factorization assumption (the result of [10] is obtained from (3.3) with the substitution $\eta_V \to k_F$). As discussed in [10], the separate predictions of a_S and a_+ involve an additional unknown coupling which is difficult to interpret in the framework of VMD.

The evaluation of the $\mathcal{O}(p^6)$ local terms generated by vector meson exchange is more involved [13, 14]. In general, we can identify two kinds of contributions: those originating from genuine $\mathcal{O}(p^6)$ weak transitions and those generated by the pole expansion of the leading $\mathcal{O}(p^4)$ VMD results (we refer to [13, 14] for a detailed discussion). Interestingly enough, all the effects of the genuine $\mathcal{O}(p^6)$ weak transitions, evaluated under the factorization assumption, drop out in the sum $W_+^{\rm pol} + W_S^{\rm pol}$. Moreover, the $\mathcal{O}(p^6)$ vector exchange contributions only modify the slope parameters b_i, leading to the simple result

$$W^{(6)}_{+,V} + W^{(6)}_{S,V} = \frac{G_8}{G_F}\,\frac{z}{r_V^2}\left[16\pi^2 f_V^2\,(\,2\eta_V - 1\,)\right]. \tag{3.4}$$

Combining (3.3) and (3.4) and neglecting the presumably small logarithmic term in (3.3), we then obtain

$$\frac{b^{(6)}_{+,V} + b^{(6)}_{S,V}}{a^{(4)}_{+,V} + a^{(4)}_{S,V}} = \frac{1}{r_V^2}\,. \tag{3.5}$$

In order to disentangle neutral and charged channels beyond the above relation one has to rely on additional assumptions. We notice that out of the two solutions for a_+

JHEP08(1998)004

and b_+ discussed in the previous section, the one with $a_+ < 0$, consistent with chiral counting, is also compatible with the relation $b_+/a_+ = 1/r_V^2$. This would follow directly from the assumption of vector meson dominance in the polynomial part of the form factor, i.e. from the hypothesis

$$W_i^{\rm pol}(z) = a_i \, \frac{r_V^2}{r_V^2 - z} \, . \tag{3.6}$$

As commented previously, both form factors $W_+^{\rm pol}$ and $W_S^{\rm pol}$ may receive short–distance type contributions that in principle could spoil this relation. However, the form factor of the matrix element $\langle \pi | \overline{s} \gamma^\mu d | K \rangle$ exhibits a clear $K^*(892)$ pole dominance (in the $SU(3)$ limit it coincides with the electromagnetic form factor $F(z)$ discussed in the previous section). Hence, we conclude that the pole structure of $W_i^{\rm pol}(z)$ is not restricted to the long–distance part. Only the genuine $\mathcal{O}(p^6)$ weak vector transitions could spoil the relation (3.6). If these were negligible, and in general this is not necessarily the case, the ratio $b_i/a_i = 1/r_V^2$ would hold for both channels.

We can use this assumption to analyze the $K_S \to \pi^0 \ell^+ \ell^-$ mode. Taking a look at the expressions of $B(K_S \to \pi^0 \ell^+ \ell^-)$ in (2.12) and considering that $a_S b_S > 0$ (as follows from $b_S/a_S = 1/r_V^2$), we observe that for $|a_S| \gtrsim 0.2$ (i.e. for $B(K_S \to \pi^0 e^+ e^-) \gtrsim 2 \times 10^{-10}$) the constant and linear terms in a_S and b_S are negligible and can be dropped. In this way we obtain an expression for the branching ratios in terms of just one parameter

$$\begin{aligned} B(K_S \to \pi^0 e^+ e^-) \;&\simeq\; 5.2 \, a_S^2 \; \times 10^{-9} \, , \\ B(K_S \to \pi^0 \mu^+ \mu^-) \;&\simeq\; 1.2 \, a_S^2 \; \times 10^{-9} \, , \end{aligned} \tag{3.7}$$

from where we predict

$$\frac{B(K_S \to \pi^0 \mu^+ \mu^-)}{B(K_S \to \pi^0 e^+ e^-)} \simeq 0.23 \, . \tag{3.8}$$

Unfortunately the lack of information on a_S does not allow us to predict the separate rates. We note however that if $a_S \sim \mathcal{O}(1)$, as expected on general grounds, the electron mode should be within reach of the KLOE experiment [5].

4. CP violation in $K \to \pi \ell^+ \ell^-$ decays

The dominance of single photon exchange does not apply to the CP–violating parts of $K \to \pi \ell^+ \ell^-$ amplitudes. In this case the hierarchy of the CKM matrix implies that Z^0–penguin and $W^\pm$–box diagrams, dominated by short–distance contributions, play an important role. As a result, CP–violating amplitudes can be more conveniently studied by means of an appropriate four–fermion Hamiltonian that, in the case of $s \to d \ell^+ \ell^-$

transitions, is known to next–to–leading order [15]. At scales $\mu < m_c$ it is given by

$$\mathcal{H}_{eff}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \Big[\sum_{i=1}^{6,7V} (z_i(\mu) + \tau y_i(\mu)) Q_i(\mu) \; + \; \tau y_{7A}(M_W) Q_{7A}(M_W) \Big] + \text{h.c.}, \quad (4.1)$$

where $\tau = -(V_{ts}^* V_{td})/(V_{us}^* V_{ud})$ and V_{ij} denote the CKM matrix elements. Here $Q_{1,2}$ are the current–current operators, $Q_3, \ldots, Q_6$ the QCD penguin operators, whereas

$$Q_{7V} \; = \; \overline{s}\gamma^\mu (1-\gamma_5) d \, \overline{\ell}\gamma_\mu \ell \qquad \text{and} \qquad Q_{7A} \; = \; \overline{s}\gamma^\mu (1-\gamma_5) d \, \overline{\ell}\gamma_\mu \gamma_5 \ell \qquad (4.2)$$

are generated by electroweak penguin and box diagrams [16, 17]. In the standard phase convention, the overall factor $V_{us}^* V_{ud}$ is chosen to be real and direct CP violation is driven by the imaginary part of τ. With this choice of phase, only the QCD penguin operators and Q_{7V}, Q_{7A} are relevant for estimating direct CP–violating amplitudes. As can be understood by looking at the corresponding matrix elements, the dominant role is played by Q_{7V}, Q_{7A} [15], with $y_{7V}(1 \text{ GeV}) \simeq 5.7 \times 10^{-3}$ and $y_{7A}(M_W) \simeq -5.3 \times 10^{-3}$ [18] (corresponding to $\overline{m}_t(m_t) = 167\,\text{GeV}$).

The contribution of Q_{7V} to $K \to \pi \ell^+ \ell^-$ decays interferes with the long–distance amplitude discussed in the previous sections. In fact, the contribution proportional to the coefficient z_{7V} is already included in the polynomial part of the $K \to \pi\gamma^*$ form factor (as discussed in section 3). On the contrary, the interference of Q_{7A} with the photon exchange amplitude vanishes as long as the lepton polarizations are summed over. In the following we shall discuss the role of $y_{7V} Q_{7V}$ in $K_L \to \pi^0 e^+ e^-$ and $K^\pm \to \pi^\pm e^+ e^-$ decays.

4.1. $K_L \to \pi^0 e^+ e^-$

From the definition (2.1), CP invariance would imply that the $K_L \to \pi^0 \gamma^*$ form factor $W_L(z)$ vanishes. In the limit of CP conservation, the decay $K_L \to \pi^0 e^+ e^-$ can only proceed through the two–photon process $K_L \to \pi^0 \gamma\gamma \to \pi^0 e^+ e^-$ [19, 20, 21] or via subleading terms in the expansion of the $W^\pm$–box diagram [22]. The former mechanism is expected to be dominant yielding $B(K_L \to \pi^0 e^+ e^-)_{CPC} \lesssim$ few $\times 10^{-12}$ [13, 20, 21].

In the presence of CP violation, the $K_L \to \pi^0 e^+ e^-$ transition can also proceed through the small $\varepsilon_K K_1$ piece of the K_L wave function (indirect CP violation) or via the direct CP–violating amplitude generated by (4.1). The contribution of $y_{7V} Q_{7V}$ appears in the function W_L^{pol}, defined in analogy to (2.7), with

$$a_L \; = -\, i \frac{4\pi}{\sqrt{2}} \frac{\text{Im}\; \lambda_t}{\alpha} y_{7V} \qquad \text{and} \qquad \frac{b_L}{a_L} \simeq \frac{1}{r_V^2}\,, \qquad (4.3)$$

where $\lambda_t = V_{td} V_{ts}^*$. On the other side, the indirect CP–violating contribution is given in terms of the parameter ε_K as $A(K_L \to \pi^0 e^+ e^-)_{\text{ind}} = |\varepsilon_K| e^{i\pi/4} A(K_S \to \pi^0 e^+ e^-)$.

JHEP08(1998)004

Summing the two vector–like CP–violating contributions, one has

$$W_L^{\rm pol} = 6.9 \times 10^{-4} \left[3.3\, a_S\, e^{i\frac{\pi}{4}} \; - \; i \frac{\text{Im } \lambda_t}{10^{-4}} \right] \left[1 + \frac{z}{r_V^2} \right]. \tag{4.4}$$

Unfortunately the sign of a_S is not known. Moreover, we recall that for $|a_S| \gtrsim 0.2$ the dominant term in $B(K_S \to \pi^0 e^+ e^-)$ is provided by a_S^2, therefore it is hopeless trying to extract information about the sign of a_S from the measurement of $B(K_S \to \pi^0 e^+ e^-)$.[5] However, since a_S is real a strong interference (constructive or destructive) is expected for $|a_S|$ and $|\text{Im } \lambda_t / 10^{-4}|$ of the same order of magnitude.

Including the CP–violating contribution of the Q_{7A} operator that does not interfere with (4.4), we collect all the CP–violating terms in $K_L \to \pi^0 e^+ e^-$ obtaining

$$B(K_L \to \pi^0 e^+ e^-)_{CPV} \; = \; \left[15.3\, a_S^2 \; - \; 6.8 \frac{\text{Im } \lambda_t}{10^{-4}}\, a_S \; + \; 2.8 \left(\frac{\text{Im } \lambda_t}{10^{-4}} \right)^2 \right] \times 10^{-12}. \tag{4.5}$$

From (3.7) and (4.5) a very interesting scenario emerges for $a_S \lesssim -0.5$ or $a_S \gtrsim 1.0$. Since $\text{Im } \lambda_t$ is expected to be $\sim 10^{-4}$, one would have $B(K_L \to \pi^0 e^+ e^-)_{CPV} \gtrsim 10^{-11}$ in this case. Then the CP–conserving contribution, which does not interfere with the CP–violating part in the rate, could be neglected. Moreover, the $K_S \to \pi^0 e^+ e^-$ branching ratio would be large enough to allow a direct determination of $|a_S|$. Thus, from the interference term in (4.5) one could perform an independent measurement of $\text{Im } \lambda_t$, with a precision increasing with the value of $|a_S|$.

A relation similar to (4.5) can be traced from the work of Dib et al. [17] (see also [18]). We note that this result arises from the VMD assumption about the $K_S \to \pi^0 e^+ e^-$ form factor discussed in Section 3, which leads to (4.4). According to this hypothesis, the relative weight of direct and indirect CP–violating components in (4.5) is not affected by possible cuts in the z spectrum. On the contrary, we recall that the CP–conserving rate does have a different (softer) spectrum in z: essentially the factor $\lambda^{3/2}(1, z, r_\pi^2)$ in (2.3) is replaced by $\lambda^{5/2}(1, z, r_\pi^2)$ [19, 21, 22]. This fact could provide an additional handle for disentangling the various components of $K_L \to \pi^0 e^+ e^-$.

4.2. Charge asymmetry

The interference of the long–distance $K \to \pi \gamma^*$ amplitude and the short–distance contribution of $y_{7V} Q_{7V}$ leads to an asymmetry between the widths of $K^+ \to \pi^+ e^+ e^-$

[5] In principle the relative sign of a_S with respect to $A(K_S \to \pi^+ \pi^- \pi^0)$ could be measured by a careful analysis of the $K_S \to \pi^0 e^+ e^-$ spectrum (similarly to a_+ in the charged mode). However, this program is made very difficult by the smallness of $A(K_S \to \pi^+ \pi^- \pi^0)$. In addition, even if it were possible, one would then have to rely on a model–dependent assumption about the sign of $A(K_S \to \pi^+ \pi^- \pi^0)$ from (4.1).

JHEP08(1998)004

and $K^- \to \pi^- e^+ e^-$, which is a clear signal of direct CP violation [19]. This quantity, defined by

$$\Gamma(K^+ \to \pi^+ e^+ e^-) \; - \; \Gamma(K^- \to \pi^- e^+ e^-) \; = \; \frac{G_F \, \alpha^2 \, M_K^3}{768 \, \pi^5} \; \times$$
$$\int_{4r_e^2}^{(1-r_\pi)^2} dz \, \lambda^{3/2}(1, z, r_\pi^2) \sqrt{1 - 4\frac{r_e^2}{z}} \left(1 + 2\frac{r_e^2}{z}\right) \mathrm{Im} \; W_+^{\mathrm{pol}}(z) \, \mathrm{Im} \; W_+^{\pi\pi}(z) \, ,$$

can be calculated unambiguously up to a sign. Indeed, the only contribution to $\mathrm{Im} \; W_+^{\mathrm{pol}}$ comes from the short–distance regime and it is given by

$$\mathrm{Im} \; a_+ \; = \; \frac{4\pi}{\sqrt{2}} \, \frac{\mathrm{Im} \; \lambda_t}{\alpha} \, y_{7V} \, , \qquad \qquad \frac{\mathrm{Im} \; b_+}{\mathrm{Im} \; a_+} \simeq \frac{1}{r_V^2} \, , \tag{4.6}$$

whereas the imaginary part of $W_+^{\pi\pi}$ is determined by the physical $K^+ \to \pi^+\pi^+\pi^-$ amplitude. Using the results in (2.8-2.10) and the experimental value of $\Gamma(K^+ \to \pi^+ e^+ e^-)$, we get

$$\frac{|\Gamma(K^+ \to \pi^+ e^+ e^-) - \Gamma(K^- \to \pi^- e^+ e^-)|}{\Gamma(K^+ \to \pi^+ e^+ e^-) + \Gamma(K^- \to \pi^- e^+ e^-)} \; = \; (0.071 \pm 0.007) \times |\mathrm{Im} \; \lambda_t| \, . \tag{4.7}$$

The numerical value in (4.7) corresponds to an increase of a factor ~ 2 over the estimate at leading order in the chiral expansion [19]. However, given that $\mathrm{Im} \; \lambda_t \sim 10^{-4}$, it is still very difficult to detect such an effect, at least within the Standard Model.

A more interesting observable is the unintegrated asymmetry defined by

$$\delta\Gamma(z) \; \equiv \; \frac{\left| \frac{d\Gamma}{dz}(K^+ \to \pi^+ e^+ e^-) \; - \; \frac{d\Gamma}{dz}(K^- \to \pi^- e^+ e^-) \right|}{\Gamma(K^+ \to \pi^+ e^+ e^-) + \Gamma(K^- \to \pi^- e^+ e^-)} \tag{4.8}$$

that we have plotted in Fig. 5 both at leading order in the chiral expansion and including unitarity corrections. This asymmetry is seen to have a maximum around $z \simeq 0.4$ that, being far away from the dominant background, could be more easily accessible from the experimental point of view than the integrated asymmetry.

5. Conclusions

We have performed a model–independent analysis of $K^+ \to \pi^+ \ell^+ \ell^-$ and $K_S \to \pi^0 \ell^+ \ell^-$ decays including $K \to 3\pi$ unitarity corrections and a general polynomial decomposition of the remaining dispersive amplitude. Using as input the presently available data on $K^+ \to \pi^+ e^+ e^-$ [6], we concluded that the ratio $R = B(K^+ \to \pi^+ \mu^+ \mu^-)/B(K^+ \to \pi^+ e^+ e^-)$ must be larger than 0.23 unless there is a contribution from some non–standard physics. This result is to be compared with the recent measurement $R = 0.167 \pm 0.036$ [7].

JHEP08(1998)004

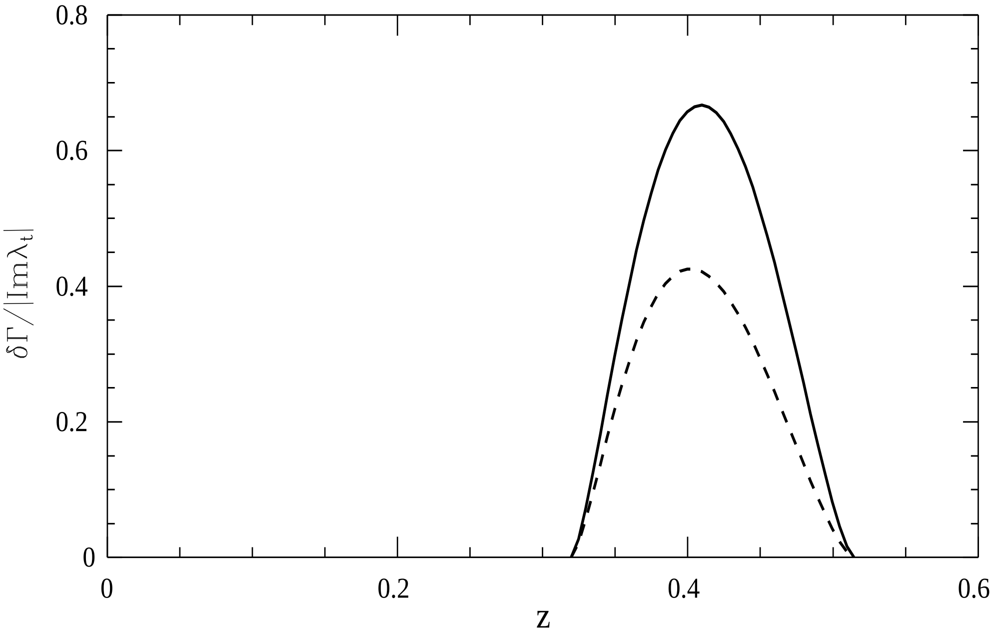

Figure 5: Differential charge asymmetry defined in (4.8). The dashed line is the leading–order result in the chiral expansion, whereas the full line includes the unitarity corrections.

We have shown that it is very difficult to predict $B(K_S \to \pi^0 e^+ e^-)$ without strong model–dependent assumptions, except for an approximate upper bound $B(K_S \to \pi^0 e^+ e^-) \lesssim 10^{-8}$. On the other hand, a realistic estimate of the muon/electron ratio can be obtained in the framework of VMD: $B(K_S \to \pi^0 \mu^+ \mu^-)/B(K_S \to \pi^0 e^+ e^-) \simeq 0.23$.

We have reanalysed the CP–violating contributions to $K_L \to \pi^0 e^+ e^-$, emphasizing a possible strong interference (destructive or constructive) between direct and indirect CP–violating amplitudes. Together with a precise determination of $B(K_S \to \pi^0 e^+ e^-)$, this could provide a possible handle for isolating the direct CP–violating component. Finally, while the unitarity corrections lead to an increase of about 100% for the charge asymmetry $\Gamma(K^+ \to \pi^+ e^+ e^-) - \Gamma(K^- \to \pi^- e^+ e^-)$, the Standard Model prediction is still too small to be within reach of forthcoming experiments.

Note added: At the recent ICHEP98 in Vancouver, preliminary results from a high–statistics $K^+ \to \pi^+ e^+ e^-$ experiment (BNL-E865, presented by Hong Ma) were announced. While the mean value of the decay rate agrees with the previous measurement [6], the slope is significantly bigger: $\lambda = 0.20 \pm 0.02$. Although such a big slope is still compatible with the theoretical expectation $|b_+| < |a_+|$, the parameter $|b_+|$ would have to be bigger than the naive VMD prediction $b_+ = a_+/r_V^2$, but consistent with the factorization model predictions for the genuine $\mathcal{O}(p^6)$ vector meson contribution. Finally, we observe that the larger slope aggravates the discrepancy between theory and experiment for the μ/e ratio R.

JHEP08(1998)004

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JHEP08(1998)004

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JHEP08(1998)004